

# Cooling distant atoms into entangled state via coupled cavities

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We propose a scheme for generating steady entanglement between two distant atomic qubits in the coupled-cavity system via laser cooling. With suitable choice of the laser frequencies, the target entangled state is the only ground state that is not excited by the lasers due to large detunings. The laser excitations of other ground states, together with dissipative processes, drive the system to the target state which is the unique steady state of the system. Numerical simulation shows that the maximally entangled state with high fidelity can be produced with presently available cooperativity.

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The major difficulty in the implementation of a quantum processor is the decoherence due to coupling to the environment. Especially for the entanglement between distant nodes, the coherence usually becomes very fragile when it needs to be manipulated externally. This problem can not be fully solved just based on unitary dynamics [1–5]. Recently, the dissipation has been used as a resource to generate the long-time entanglement [6–17]. One recent experiment has demonstrated the dissipative preparation of entangled steady-state [18]. Approaches based on this idea do not require photon detection, observation of macroscopic fluorescence signals, and definite control of initial state and evolution time, which are very robust against moderate environment noise. Vacanti and Beige [6] showed the possibility of preparing highly entangled states by methods being close analogy to laser sideband cooling. Busch *et al.* [7] proposed an improved entanglement cooling scheme for two atoms inside an optical cavity with a rather low cooperative parameter. We have proposed a scheme for producing entangled states between two atoms trapped in two coupled cavities in the steady state [17]. The scheme is based on suppression of the effective decay of the target entangled state induced by cavity loss by suitably setting the cavity detuning.

In this paper, we propose an alternative scheme to prepare the highly entangled steady-state for two atoms trapped in two coupled cavities based on the idea of laser cooling. Unlike the scheme of Ref. [17], the target entangled state becomes the steady state due to the suppression of its laser excitation. Through suitable choice of the laser frequencies, the target entangled state is not affected by the classical drivings due to large detunings, while each of the other ground states are resonantly coupled to the corresponding dressed excited states of the whole atom-cavity system by one laser field. Each excited state decays to the target state and other ground states due to dissipative dynamics. As a result, the target state is the unique steady state. We find a distinct improvement in the fidelity compared with the previous

scheme [17, 19]. The distributed entangled steady-state with the fidelity above 90% can be obtained after a moderate evolution time even when the cooperative parameter is only 50, which is impossible by previous methods [17, 19]. This present work may constitute an important step toward the realization of quantum networks with current experimental technology for the coupled-cavity system [20, 21].

The experimental setup consists of two identical  $\Lambda$ -type atoms trapped in two directly coupled cavities respectively, as shown in Fig. 1. The level  $|i\rangle$  of each atom has the corresponding energy  $w_i$  ( $i = 0, 1, 2$ ). Without optical laser driving, the Hamiltonian of the whole system can be written as:

$$H_{NL} = \sum_{j=1}^2 \sum_{i=1}^2 w_i |i\rangle_{jj} \langle i| + \sum_{j=1}^2 w_a a_j^\dagger a_j + J(a_1^\dagger a_2 + a_1 a_2^\dagger) + \sum_{j=1}^2 g(|2\rangle_{jj} \langle 1| a_j + |1\rangle_{jj} \langle 2| a_j^\dagger), \quad (1)$$

where  $a_j$  and  $a_j^\dagger$  are the annihilation and creation operators for the  $j$ th cavity field mode with frequency  $w_a$  respectively. We here have set the energy  $w_0$  of level  $|0\rangle$  to be zero. The  $j$ th cavity field mode resonantly couples to the  $|1\rangle_j \leftrightarrow |2\rangle_j$  transition with coupling constant  $g$ , i.e.,  $w_a = w_2 - w_1$ .  $J$  is the photon hopping strength between two coupled cavities. We obtain the associated eigenstate and eigenenergy within zero-excitation subspace and one-excitation subspace analytically, as shown in Table I and Table II. The notation  $|AB, CD\rangle$  represents that atom 1 (2) is in the state  $|A\rangle$  ( $|B\rangle$ ) and there are  $C$  ( $D$ ) photons in cavity 1 (2). Due to the coherent photon hopping between two cavities, the eigenenergies in the one-excitation subspace are shifted. The corresponding eigenstates are

$$\begin{aligned} |\phi_1\rangle &= \frac{1}{N_a} \left( \frac{\sqrt{J^2 + g^2}}{g} |10, 10\rangle + \frac{J}{g} |10, 01\rangle + |20, 00\rangle \right), \\ |\phi_2\rangle &= \frac{1}{N_a} \left( \frac{J}{g} |01, 10\rangle + \frac{\sqrt{J^2 + g^2}}{g} |01, 01\rangle + |02, 00\rangle \right), \\ |\phi_3\rangle &= \frac{1}{N_a} \left( \frac{J}{g} |10, 01\rangle - \frac{\sqrt{J^2 + g^2}}{g} |10, 10\rangle + |20, 00\rangle \right), \end{aligned}$$

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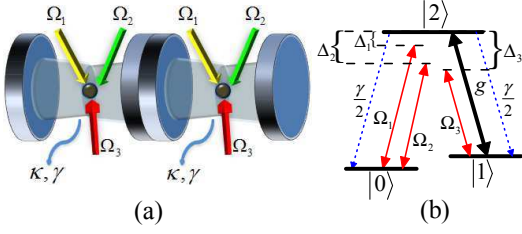


FIG. 1. (Color online) (a) Experimental schematic for cooling two identical  $\Lambda$ -type atoms into a maximally entangled state via two directly coupled cavities.  $\gamma$  and  $\kappa$  are the atomic spontaneous emission and the cavity decay, respectively. (b) Each atomic level configuration with three off-resonant lasers.  $\Omega_k$  ( $k = 1, 2, 3$ ) is the  $k$ th laser with relevant detuning  $\Delta_i$ .

TABLE I. The eigenstates and eigenenergies of the Hamiltonian in Eq. (1) within the zero-excitation subspace.

Eigenstate	Eigenenergy
$ \phi_{00}\rangle =  00, 00\rangle$	0
$ T, 00\rangle = \frac{1}{\sqrt{2}}( 01, 00\rangle +  10, 00\rangle)$	$w_1$
$ S, 00\rangle = \frac{1}{\sqrt{2}}( 01, 00\rangle -  10, 00\rangle)$	$w_1$
$ \phi_{11}\rangle =  11, 00\rangle$	$2w_1$

$$\begin{aligned}
 |\phi_4\rangle &= \frac{1}{N_a} \left( \frac{J}{g} |01, 10\rangle - \frac{\sqrt{J^2 + g^2}}{g} |01, 01\rangle + |02, 00\rangle \right), \\
 |\phi_5\rangle &= \frac{1}{N_b} \left( -\frac{g}{J} |10, 01\rangle + |20, 00\rangle \right), \\
 |\phi_6\rangle &= \frac{1}{N_b} \left( -\frac{g}{J} |01, 10\rangle + |02, 00\rangle \right), \\
 |\phi_7\rangle &= \frac{1}{N_c} (|00, 10\rangle - |00, 01\rangle), \\
 |\phi_8\rangle &= \frac{1}{N_c} (|00, 10\rangle + |00, 01\rangle), \\
 |\phi_9\rangle &= \frac{1}{N_d} \left[ \frac{J - \sqrt{J^2 + 4g^2}}{2g} (|11, 10\rangle - |11, 01\rangle) \right. \\
 &\quad \left. - |21, 00\rangle + |12, 00\rangle \right], \\
 |\phi_{10}\rangle &= \frac{1}{N_e} \left[ \frac{J + \sqrt{J^2 + 4g^2}}{2g} (|11, 10\rangle - |11, 01\rangle) \right. \\
 &\quad \left. - |21, 00\rangle + |12, 00\rangle \right],
 \end{aligned}$$

TABLE II. The eigenstates and eigenenergies of the Hamiltonian in Eq. (1) within the one-excitation subspace.

Eigenstate	Eigenenergy
$ \phi_1\rangle,  \phi_2\rangle$	$\lambda_1 = \lambda_2 = w_2 + \sqrt{J^2 + g^2}$
$ \phi_3\rangle,  \phi_4\rangle$	$\lambda_3 = \lambda_4 = w_2 - \sqrt{J^2 + g^2}$
$ \phi_5\rangle,  \phi_6\rangle$	$\lambda_5 = \lambda_6 = w_2$
$ \phi_7\rangle$	$\lambda_7 = w_2 - w_1 - J$
$ \phi_8\rangle$	$\lambda_8 = w_2 - w_1 + J$
$ \phi_9\rangle$	$\lambda_9 = w_1 + w_2 - J/2 + \sqrt{J^2 + 4g^2}/2$
$ \phi_{10}\rangle$	$\lambda_{10} = w_1 + w_2 - J/2 - \sqrt{J^2 + 4g^2}/2$
$ \phi_{11}\rangle$	$\lambda_{11} = w_1 + w_2 + J/2 - \sqrt{J^2 + 4g^2}/2$
$ \phi_{12}\rangle$	$\lambda_{12} = w_1 + w_2 + J/2 + \sqrt{J^2 + 4g^2}/2$

$$\begin{aligned}
 |\phi_{11}\rangle &= \frac{1}{N_d} \left[ \frac{J - \sqrt{J^2 + 4g^2}}{2g} (|11, 10\rangle + |11, 01\rangle) \right. \\
 &\quad \left. + |21, 00\rangle + |12, 00\rangle \right], \\
 |\phi_{12}\rangle &= \frac{1}{N_e} \left[ \frac{J + \sqrt{J^2 + 4g^2}}{2g} (|11, 10\rangle + |11, 01\rangle) \right. \\
 &\quad \left. + |21, 00\rangle + |12, 00\rangle \right],
 \end{aligned} \tag{2}$$

where  $N_a = \frac{\sqrt{2(J^2 + g^2)}}{g}$ ,  $N_b = \frac{\sqrt{J^2 + g^2}}{J}$ ,  $N_c = \frac{1}{\sqrt{2}}$ ,  $N_d = \sqrt{\frac{1}{g^2}(J^2 + 4g^2 - J\sqrt{J^2 + 4g^2})}$ ,  $N_e = \sqrt{\frac{1}{g^2}(J^2 + 4g^2 + J\sqrt{J^2 + 4g^2})}$ . To cool the atoms into the maximally entangled state  $|T\rangle$ , three optical lasers are simultaneously applied to each atom. We suppose the  $|0\rangle \leftrightarrow |2\rangle$  transition of each atom is driven by two lasers with Rabi frequencies  $\Omega_m$  and frequencies  $w_{L,m}$  ( $m = 1, 2$ ), while the  $|1\rangle \leftrightarrow |2\rangle$  transition of each atom is driven by another laser with Rabi frequency  $\Omega_3$  and frequencies  $w_{L,3}$ . Under the rotating wave approximation, the interaction Hamiltonian between the atoms and lasers are described as:

$$\begin{aligned}
 H_{AL} &= \sum_{j=1}^2 \sum_{m=1}^2 (\Omega_m e^{i w_{L,m} t} |0\rangle_{jj} \langle 2| + H.c.) \\
 &\quad + \sum_{j=1}^2 (\Omega_3 e^{i w_{L,3} t} |1\rangle_{jj} \langle 2| + H.c.).
 \end{aligned} \tag{3}$$

Under the weak excitation condition, the probability that the system is excited to the subspaces with more than one excitation can be neglected. The laser-atom interaction Hamiltonian in Eq. 3 can be expanded in terms of the eigenstates in Table I and Table II:

$$\begin{aligned}
 H'_{AL} &= e^{i H_{NL} t} H_{AL} e^{-i H_{NL} t} \\
 &= \sum_{x=1}^2 \Omega_x \left[ \frac{\sqrt{2}g}{2} L_1 \sum_{k=1}^4 e^{i(w_{L,x} - \lambda_k)t} |00, 00\rangle \langle \phi_k| \right. \\
 &\quad + J L_1 \sum_{k=5}^6 e^{i(w_{L,x} - \lambda_k)t} |00, 00\rangle \langle \phi_k| \\
 &\quad + L_2 e^{i(w_{L,x} + w_1 - \lambda_9)t} |S, 00\rangle \langle \phi_9| \\
 &\quad - L_3 e^{i(w_{L,x} + w_1 - \lambda_{10})t} |S, 00\rangle \langle \phi_{10}| \\
 &\quad - L_2 e^{i(w_{L,x} + w_1 - \lambda_{10})t} |T, 00\rangle \langle \phi_{11}| \\
 &\quad \left. + L_3 e^{i(w_{L,x} + w_1 - \lambda_{12})t} |T, 00\rangle \langle \phi_{12}| \right] \\
 &\quad + \Omega_3 \left[ \frac{g}{2} L_1 \sum_{k=1}^4 e^{i(w_{L,3} + w_1 - \lambda_k)t} |T, 00\rangle \langle \phi_k| \right. \\
 &\quad + \frac{J}{\sqrt{2}} L_1 \sum_{k=5}^6 e^{i(w_{L,3} + w_1 - \lambda_k)t} |T, 00\rangle \langle \phi_k| \\
 &\quad + \frac{g}{2} L_1 \sum_{k=1}^4 (-1)^k e^{i(w_{L,3} + w_1 - \lambda_k)t} |S, 00\rangle \langle \phi_k| \\
 &\quad + \frac{J}{\sqrt{2}} L_1 \sum_{k=5}^6 (-1)^k e^{i(w_{L,3} + w_1 - \lambda_k)t} |S, 00\rangle \langle \phi_k| \\
 &\quad \left. - L_2 e^{i(w_{L,3} + 2w_1 - \lambda_{11})t} |11, 00\rangle \langle \phi_{11}| + \right.
 \end{aligned}$$

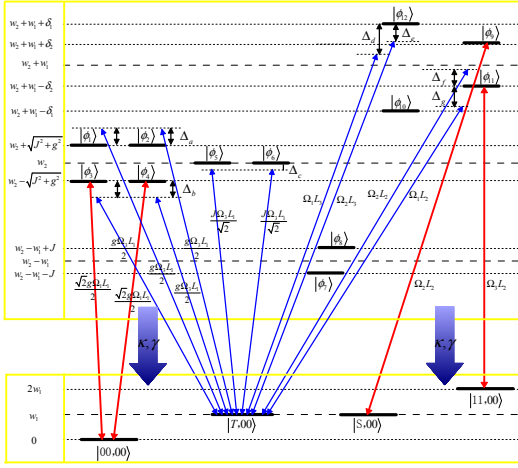


FIG. 2. (Color online) Level configuration in the dressed state picture, exhibiting resonant transitions for ground states  $|00,00\rangle$ ,  $|S,00\rangle$  and  $|11,00\rangle$  by tuning three appropriate laser frequencies, and the possible transitions induced by all the lasers for target state  $|T,00\rangle$  between the zero-excitation subspace and one-excitation subspace. All the Rabi frequencies and detunings of those transitions are obviously marked, where  $\delta_1 = \sqrt{J^2 + 4g^2}/2 + J/2$ ,  $\delta_2 = \sqrt{J^2 + 4g^2}/2 - J/2$ ,  $\Delta_a = J/2 - \sqrt{J^2 + 4g^2}/2 - \sqrt{J^2 + g^2}$ ,  $\Delta_b = J/2 - \sqrt{J^2 + 4g^2}/2 + \sqrt{J^2 + g^2}$ ,  $\Delta_c = J/2 - \sqrt{J^2 + 4g^2}/2$ ,  $\Delta_d = -J/2 - \sqrt{J^2 + 4g^2}/2 - \sqrt{J^2 + g^2}$ ,  $\Delta_e = -J$ ,  $\Delta_f = -J + \sqrt{J^2 + 4g^2}$ ,  $\Delta_g = -J/2 + \sqrt{J^2 + 4g^2}/2 - \sqrt{J^2 + g^2}$ . The solid arrows represent one-excitation subspace dissipates into the zero-excitation subspace induced by the cavity decay  $\kappa$  and the atomic spontaneous emission  $\gamma$ .

$$L_3 e^{i(w_{L,3} + 2w_1 - \lambda_{12})t} |11,00\rangle \langle \phi_{12}| \Big] + H.c., \quad (4)$$

where  $L_1 = \frac{1}{\sqrt{J^2 + g^2}}$ ,  $L_2 = \frac{2g^2 N_d}{\sqrt{2J^2 + 8g^2(J - \sqrt{J^2 + 4g^2})}}$ ,  $L_3 = \frac{2g^2 N_e}{\sqrt{2J^2 + 8g^2(J + \sqrt{J^2 + 4g^2})}}$ . The dynamics of open dissipative system in Lindblad form is described by the master equation

$$\begin{aligned} \dot{\rho} = & -i[(H_{NL} + H_{AL}), \rho] \\ & + \frac{\kappa}{2} \sum_{j=1}^2 (2a_j \rho a_j^\dagger - a_j^\dagger a_j \rho - \rho a_j^\dagger a_j) \\ & + \frac{\gamma}{4} \sum_{j=1}^2 \sum_{k=0}^1 (2S_{jk}^- \rho S_{jk}^+ - S_{jk}^+ S_{jk}^- \rho - \rho S_{jk}^+ S_{jk}^-), \end{aligned} \quad (5)$$

where  $S_{jk}^+ = |2\rangle_j \langle k|$  and  $S_{jk}^- = |k\rangle_j \langle 2|$ . The master equation (5) is solved in the subspace spanned by the eigenstates in Table I and Table II. The fundamental physics behind the laser cooling process is the competition between the unitary dynamics induced by the optical lasers and the collective decays induced by the dissipation. Each ground state can be driven to the excited states by the laser fields, which would decay to ground states due to dissipation. The laser frequencies are suitably chosen so that transition between the target state

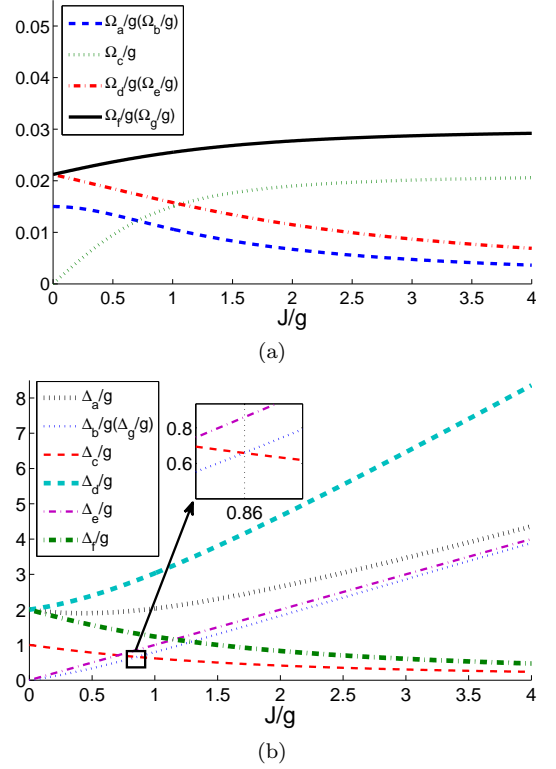


FIG. 3. (Color online) The Rabi frequencies  $\Omega_x$  ( $x = a, b, c, d, e, f, g$ ) with relevant detuning  $\Delta_x$  of the transitions for the target state  $|T,00\rangle$  is corresponding to that in Fig. 2. Different curves versus  $J/g$ : (a)  $\Omega_x/g$ ; (b)  $\Delta_x/g$ .

$|T,00\rangle$  and each excited state of the atom-cavity system is far off-resonant with all the applied lasers, while each of the other ground states  $|00,00\rangle$ ,  $|S,00\rangle$  and  $|11,00\rangle$  is resonantly coupled to at least one excited state through the laser fields. This ensures the target state to be the unique steady state.

Assume we choose  $w_{L,1} = w_2 - \sqrt{J^2 + g^2}$ ,  $w_{L,2} = w_2 - J/2 + \sqrt{J^2 + 4g^2}/2$ ,  $w_{L,3} = w_2 - w_1 + J/2 - \sqrt{J^2 + 4g^2}/2$ . When the condition  $w_1 \gg J + \sqrt{J^2 + g^2}$ ,  $J/2 + \sqrt{J^2 + 4g^2}/2 + \sqrt{J^2 + g^2}$  is satisfied, the detunings for the laser-driven transitions between the target state  $|T,00\rangle$  and excited states in Table II depend on the atom-cavity coupling strength  $g$  and the cavity-cavity hopping strength  $J$ , as shown in Fig. 2. From the results of Fig. 3, when  $J$  is within  $0.8g \sim 1.5g$ , all the relevant detunings are much larger than the corresponding Rabi frequencies which means the excitation of the target state  $|T,00\rangle$  is highly suppressed, while the populations of the other ground states are quickly transferred to the excited states by resonant laser excitations, followed by decay into the target state  $|T\rangle$  and other ground states due to dissipative dynamics. This means that  $|T,00\rangle$  is the unique steady state. Based on the above conditions, a set of the optimal parameters is obtained by numerically solving the full master equation (5), as shown in Fig. 4. The influences of fluctuations in  $J$  and Rabi fre-

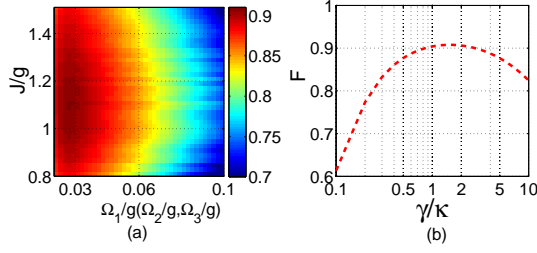


FIG. 4. (Color online) (a) The population of the state  $|T\rangle$  versus the photon hopping strength and the lasers strength at a finite evolution time with  $C = 50$  and  $\gamma = 2\kappa$ . (b) The fidelity of state  $|T\rangle$  versus different ratios  $\gamma/\kappa$  with  $C = 50$ ,  $w_1 = 8g$ ,  $w_2 = 18g$ ,  $w_a = w_2 - w_1$ ,  $J = 1.1g$ ,  $\Omega_1 = \Omega_2 = \Omega_3 = 0.03g$  and  $gt = 1500$ .

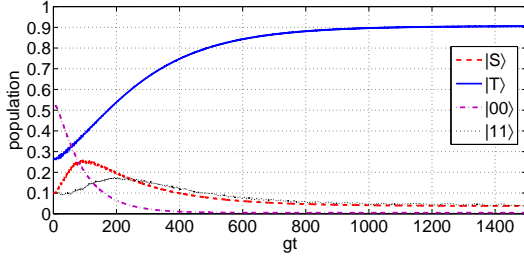


FIG. 5. (Color online) The populations of different ground states versus the system evolution  $gt$  with an arbitrary initial state, where  $C = 50$ ,  $\gamma = 2\kappa$  and other parameters are the same as those in Fig. 4(b).

quencies on the fidelity are considered in Fig. 4. Even when there is 10% fluctuations in  $J$ , the fidelity of state  $|T\rangle$  only decreases 1%. However, the fidelity is sensitive to the fluctuations in Rabi frequencies. The influence of different ratios  $\gamma/\kappa$  on the fidelity is shown in Fig. 4(b), and the optimal ratio appears near  $\gamma = 1.5\kappa$ . For this set of optimized experiment parameters, the populations of different ground states versus  $gt$  are plotted in Fig. 5. The numerical results show that the target steady-state can be obtained with high fidelity even for low cooperativity  $C \sim 50$ , which is experimentally accessible [22]. This is a significant improvement as compared to previous proposals [17, 19]. In conclusion, we have proposed a feasible scheme for producing maximally entangled states for two atoms trapped in two coupled optical cavities in the steady state through laser cooling. Our method allows for a significant improvement of the fidelity as compared to the previous methods for preparation of distributed entanglement [17]. L.T.S., X.Y.C., H.Z.W., and S.B.Z. acknowledge support from the Major State Basic Research Development Program of China under Grant No. 2012CB921601, National Natural Science Foundation of China under Grant No. 10974028, the Doctoral Foundation of the Ministry of Education of China under Grant No. 20093514110009, and the Natural Science Foundation of Fujian Province under Grant No. 2009J06002. Z.B.Y. acknowledges support from the National Basic Research Program of China under Grants No. 2011CB921200 and No. 2011CBA00200, and the China Postdoctoral Science Foundation under Grant No. 20110490828.

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